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Numerical Simulation of Rigid Blocks Subjected to Rocking Motion

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ABSTRACT: This paper addresses the numerical modelling of rigid blocks subjected to rocking motion. Two different tools are considered. The first tool is analytical and overcomes the usual limitations of the traditional piecewise equations of motion through Lagrangian formalism. The second tool is based on the Discrete Element Method (DEM), especially effective for the numerical modelling of rigid blocks. An extensive comparison between numerical and experimental data has been carried out to validate and define the limitations of the analytical tools under study.

1 INTRODUCTION

Tensile resistance of historical constructions formed by large stone blocks is mainly due to their own weight which turns these structures into particularly vulnerable objects under lateral seismic loading. Due to this fact, the study based upon the assumption of continuum structures would not be realistic for many cases. On the other hand, models based on rigid-block assemblies provide a suitable framework for understanding their dynamical behaviour under seismic actions. In this context, the problem is primarily concerned with Rocking Motion (RM) dynamics (Augusti and Sinopoli 1992).

The reference analytical frame for the study of RM dynamics remains based on the formulation introduced by Housner (1963), which will be referred as classical theory in the present study. This method tackles the dynamical problem with two piecewise equations of motion for each sign of the rocking angle, while damping is reproduced by means of a coefficient of restitution. Nevertheless, Housner's classical theory presents the following drawbacks: a) the theory makes the application of standard mathematical theorems and techniques form the theory of differential equations very difficult, b) its generalization to a higher number of blocks becomes intractable when the number of degrees-of-freedom increases.

The present paper describes the numerical modelling of the experimental tests carried out on single rigid blocks, which were addressed in Peña et al. (2006a, sub). An extensive experimental investigation has been carried out to study the rocking response of four blue granite stones with different geometrical characteristics (Table I) under free vibration, harmonic and random motions of the base.

Two different tools for the numerical simulations of the rocking motion of rigid blocks are considered. The first tool, the Complex Coupled Rocking Rotations Method (CCRR), is analytical and overcomes the usual limitations of the traditional piecewise equations of motion through Lagrangian formalism. The second tool is based on the Discrete Element Method (DEM), especially effective for the numerical modelling of rigid blocks.

2 NUMERICAL MODELS

In this section a brief review of the formulation of the numerical models is presented. The complete CCRR formulation can be found in Prieto and Lourenço (2005); while the description of DEM was taken from Itasca (2000). A MatLab code was developed by the authors according to Prieto and Lourenço (2005) and the commercial UDEC code (Itasca 2000), based on the DEM, have been used to perform the numerical analyses.

2.1 Complex Coupled Rocking Rotations Method

In free rocking motion symmetry in the system is present. The symmetry is equivalent to invariance with respect to the sign of rocking angle θ, and as consequence, an associated conserved magnitude exists. This fact can be exploited by expressing θ as complex quantity, with a module r and a phase ψ, by means of:

$$\theta = re^{i\psi} \tag{1}$$

Here, the sign of θ is only associated with ψ and r equals the absolute value of θ. Therefore, the real Lagrangian function is:

$$L_0 = \frac{I}{2}\left(r'^2 + r^2\psi'^2\right) - MgR\cos(\alpha - r) \tag{2}$$

where I is the inertia moment about points O or O', g is the acceleration of the gravity and M is the mass of the block; while α and R are geometrical parameters, for a rectangular block they are defined as (Fig. 1a):

$$\alpha = \tan^{-1}\left(\frac{b}{h}\right); \quad R = \sqrt{b^2 + h^2} \tag{3}$$

This Lagrangian function constitutes, in fact, a model for free RM. The impact and seismic actions are introduced as generalized forces in the D'Alambert equations of motion (details expressions are reported in Prieto and Lourenço 2005), that they depend on the parameter p and the coefficient of restitution μ, defined as:

$$p = \sqrt{\frac{3g}{4R}}; \quad \mu = 1 - \frac{3}{2}\sin^2(\alpha) \tag{4}$$

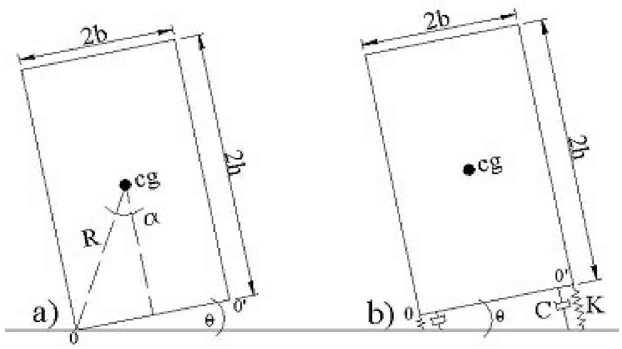

Figure 1 : Single rigid block model; (a) Classical and CCRR formulation, (b) DE model.

2.2 Discrete Element Model

The Discrete Element Model (DEM) can be considered as a method for modelling discontinuous media. This analysis technique allows relative motion between elements, which is especially suitable for problems in which the relative motion between blocks is a significant part of the deformation. It allows large displacements and rotations between blocks, including complete detachment of the blocks and it automatically detects new contacts as the calculation progresses.

The joint between two blocks is defined as two contact points located at each corner of the contact side (Fig. 1b). Each contact point is defined by means of one axial spring and one shear spring with the addition of a viscous damper. Axial spring is linear-elastic in compression and no-tension behaviour is assigned. The shear spring is linear-elastic-perfectly plastic and Coulomb-type behaviour is assumed. The viscous damping *C* is regarded as a mass *M* and stiffness *K* dependent quantity by means of the Rayleigh formulation.

Therefore the parameters required in the DE model are: axial *K* and shear *Ks* stiffness, cohesion c and friction angle φ, as well as the damping parameters ξ_{min} and f_{min}. These parameters are obtained by means of (Peña et al. 2006b):

$$K = \frac{M}{2}\left(p^2 + \frac{3g}{4R}\right); \quad Ks = K; \quad \xi_{\min} = 2\pi b f_{\min}; \quad f_{\min} = p \tag{5}$$

where *b* is the stiffness proportional damping constant defined as:

$$\sqrt{b} = 0.0057 \ln\left(\frac{KR}{\mu^2}\right) - 0.0336 \tag{6}$$

3 EXPERIMENTAL TESTS

The numerical models proposed here have been verified by comparing its response with results obtained from an extensive experimental test on the rocking response of RB. The tests were performed at the shaking table of the National Laboratory of Civil Engineering (LNEC) of Portugal on four single RB. The complete description of the experimental tests can be found in Peña et al. (2006, sub).

Each stone has different geometrical dimensions (Table 1). The dimensions of the single specimens 1, 2 and 3 were fixed to achieve a Height-With ratio (h/b) of 4, 6 and 8, respectively. In addition, single specimen number 4 was specifically manufactured with a different geometry in order to compare its dynamical performance with the rest of the stones. For this purpose, a large 45 degree cut (40 mm) at the base (Fig. 2) was performed.

In order to avoid non-desirable contact effects, a foundation of the same material was used as the base where the blocks are free to rock. This foundation was fixed to the shaking table by means of four steel bolts.

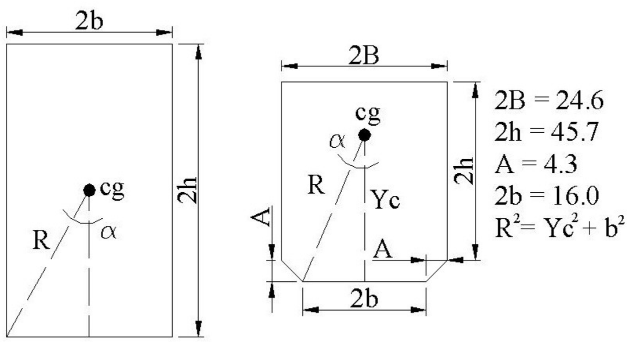

Figure 2 : Test specimens; (a) Single blocks, (b) Bi-block structure, (c) Dolmen.

Table 1 : Test specimens dimensions

Specimen	Width *2b* (m)	Height *2h* (m)	Thickness *2t* (m)	Mass *M* (kg)
1	0.25	1.000	0.754	503
2	0.17	1.000	0.502	228
3	0.12	1.000	0.375	120
4	0.16	0.457	0.750	245
Base	1.00	0.250	0.750	500

The main data obtained from the experimental tests consist of rotations around Y and Z axes, and linear displacements X and Y (see Fig. 3 for the reference coordinate system). Rotations around Y and Z were directly measured by means of a mirror linked to the blocks surface on the West face of the specimens. Two accelerometers were placed at the top of each block. One tri-axial accelerometer was located in the North face and one biaxial accelerometer was located in the south face. The displacements and accelerations of the shaking table were also measured.

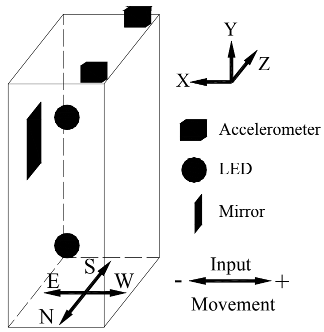

Figure 3 : Reference system of the data acquisition system and typical location of LEDs in the RB.

In order to study the dynamic behaviour of the RB, three different tests were made: a) Free rocking motion, b) Harmonic motion, and c) Random motion. The purpose of the first type of tests was twofold; the identification the parameters used in the classical theory and the calibration of the analytical models.

Whereas harmonic tests allowed to study through a simple way the dynamic behaviour of single blocks undergoing RM regime, the behaviour of the RB under earthquake conditions was analyzed by means of random test.

Thirty synthetic earthquakes compatible with the design spectrum proposed by the Eurocode 8 (2004) were generated. In order to identify them, they were labelled consecutively with an integer corresponding to their generating number. The constant branch of the spectrum is located between 0.1 and 0.3 seconds, with a spectral acceleration of 7 m/s^2 while the maximum ground acceleration is 2.8 m/s^2. The main aim of the study is to address stability of RM under random motion.

4 CALIBRATION OF THE MODELS

4.1 Theoretical and experimental parameters

The parameter set (α, p, μ) provides information of the characteristics of the RM. Their theoretical values can be obtained by Eqs. (3) and (4), or by means of free rocking motion tests (experimental values). It is well known that the experimental values of these parameters are not equal to the theoretical values, because the hypotheses assumed are not fully fulfilled. So that, the experimental parameters are adjusted by means of a minimized error surface (Peña et al. sub). In general, the experimental value of α is lower than theoretical while the restitution coefficient μ is larger. Parameter p does not show any defined pattern.

Table 2 : Theoretical and experimental classic parameters

Specimen	α (rad)		μ		p (1/s)	
	T	E	T	E	T	E
1	0.242	0.235	0.914	0.936	3.78	3.84
2	0.168	0.163	0.958	0.973	3.81	4.05
3	0.119	0.154	0.978	0.978	3.82	3.61
4	0.310	0.268	0.860	0.927	5.16	5.02

T = Theoretical, E = Experimental

Table 2 shows the theoretical and experimental values of these parameters. Experimental parameters for specimens 1 and 2 are similar to the theoretical values, with differences smaller than 3%. On the other hand, specimens 3 and 4 present significant differences in their parameters see Peña et al. (sub) for a detail discussion.

The "α" parameter can be defined as the relationship between the rotation points of the base and the position of the gravity centre of the block. The analytical models assume these rotation points as located at the block corners. However, this is true only if the block and its base are completely rigid. In practice, the rotation points are not located at the corners since the material is not perfectly rigid. In the DE model, the rotation points are located at the corners of the blocks. Thus, it is necessary to consider an equivalent base (b_{eq}), in order to take into account the real value of α parameter, defined as:

$$b_{eq} = h\tan(\alpha) \tag{7}$$

4.2 Free rocking motion

The free rocking motion tests were used to calibrate the models. The numerical models are very sensitive to variations on classical parameters, in particular to parameter α. The fitting parameters show variations smaller than 5% (Table 2). However, these small variations induce large differences in the response of the numerical models due to the high nonlinearity of the problem (Prieto and Lourenço).

Fig. 4 shows the comparison on typical free rocking tests for the responses obtained from both models using the theoretical parameters. The friction angle φ is considered equal to 30º for the four specimens. Table 3 shows the values of the parameters used in the DE model.

It can be seen that the response obtained with the theoretical parameters do not fully agree with the experimental data. In particular, the differences lie inside the period and amplitude range of each cycle. On the other hand, good agreement has been achieved between numerical and experimental results when fitted parameters are used (Fig. 5).

Table 3 : Parameters used in the definition of the DE models

Specimen	K (N/m)		ξ_{min} (x10^{-3})		f_{min} (Hz)		$2b_{eq}$ (m)	
	T	F	T	F	T	F	T	F
1	7233	6560	2.92	4.33	3.78	3.84	0.25	0.24
2	3302	3449	1.75	2.34	3.81	4.05	0.17	0.16
3	1174	2168	5.00	0.73	3.82	3.61	0.12	0.15
4	6454	6471	1.50	3.09	5.16	5.02	0.16	0.14

T = Theoretical, E = Experimental

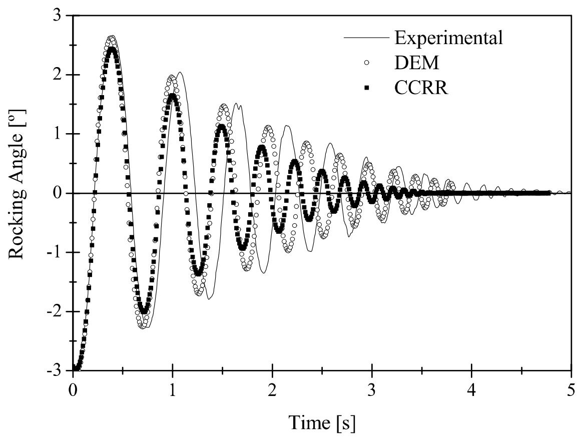

Figure 4 : Typical free rocking motion response of DE and CCRR models using theoretical parameters; Specimen 1.

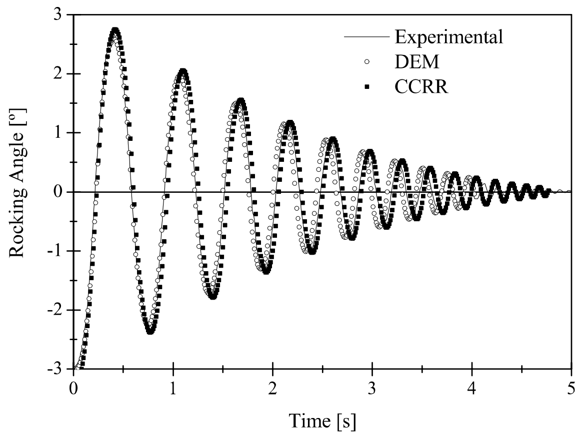

Figure 5 : Typical free rocking motion response of DE and CCRR models using fitting parameters; Specimen 1.

5 NUMERICAL SIMULATIONS

In this section, the numerical simulations carried out with the DE and CCRR models are presented. Harmonic and random excitations were used in order to verify the proposed models. The models use the parameters obtained in Section 4. These values remain unchanged for all the harmonic and random simulations.

Fig. 6 shows a typical response of RB under constant sine base motion. In order to highlight the main characteristics of the response, there is a gap in the time axis of the figure corresponding to the first part of the stationary state. The response of the model is almost the same as the experimental test even during the last cycles. The three states of harmonic motions (transient, stationary and free rocking) are well reproduced by the models.

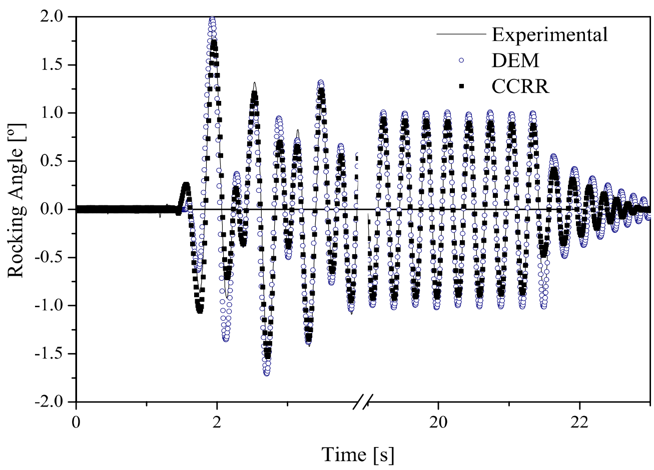

Figure 6 : Typical results of harmonic motion simulation with a constant sine excitation; specimen 1, constant sine with frequency of 3.3 Hz and amplitude of 6 mm.

It has been found that under random motion regime the response is very sensitive to perturbations in the boundary conditions as well as impact and base motion characteristics (frequency and amplitude). In particular, small changes in the initial conditions or geometrical variations due to the continuous degradation of the material at impact have shown to cause large differences in the experimental response.

Fig. 7 shows a typical result of random motion with specimen 3. The numerical models are again in good agreement with the experimental test as far as they are successful in predicting the collapse of the specimens.

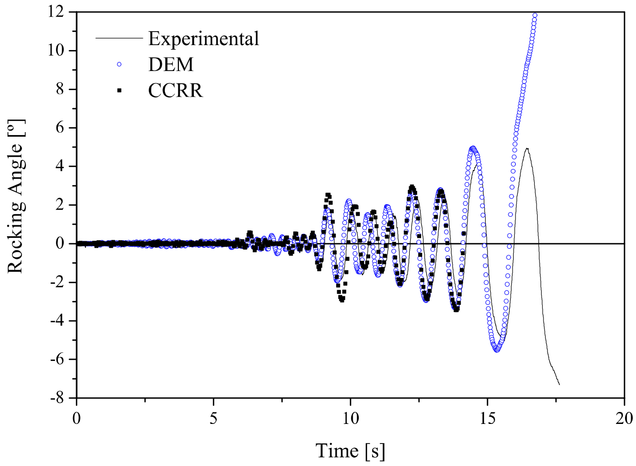

Figure 7 : Typical results of random motion tests; specimen 3, earthquake record 18 and load factor of 0.5.

6 CONCLUSIONS

This paper describes the numerical models used to study the rocking motion response of the experimental tests reported in Peña et al. (2006). Two different analytical tools were successfully calibrated and validated with the experimental results; the Complex Coupled Rocking Rotations Method (CCRR) and the Discrete Element Method (DEM), which are useful tools in the study of the rocking motion.

Both models are extremely sensitive to the classical parameters and small variations in their values produce large differences in the response. This is particularly evident for the parameter α, which is related to large geometry of the rocking block. Therefore, fitting parameters were used in order to obtain good agreement between numerical models and experimental results.

A mechanical noise introduced by the shaking table to the base motion was detected and this has a strong effect on the response of the blocks subjected to random motion. This additional source does not affect the response of the block subjected to harmonic base motion. The high frequencies introduced have shown to induce no effect into the rocking or non-rocking state of the block. However, the amount of extra mechanical energy added to the system turns the block into a more vulnerable object to lateral forces. In practice, this means that the elements subjected to a high frequency vibration will be more vulnerable to seismic effects.

Despite the limitations and difficulties to reproduce the initial and boundary conditions, good agreement has been found between numerical and experimental responses for both free and forced regimens. For random vibrations, stochastic analysis seems to be required, as indicated by the lack of repeatability of the experimental results.

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